Lecture 12

Rate-Distortion Theory and Mutual Information

Quiz Q1

You are given samples from a Uniform random variable $$U \sim\text{Unif}[0, 1]$$. We will quantize samples from the random variable $$U$$ under mean-square error (MSE) distortion criteria in the following questions.

Q1.1

What are the optimal quantization levels (intervals and reconstruction points) for a scalar one-bit quantizer under MSE distortion?
Ans: The optimal quantization levels are $\mathscr{C} = \{0.25, 0.75\}$ and the corresponding intervals are $S_1 = [0, 0.5]$ and $S_2 = (0.5, 1]$.

Quiz Q1

Q1.2

What is the average distortion per-symbol under the optimal scalar quantization scheme described above?

Ans: The average distortion per-symbol is

$$D = \int_0^{0.5} (x - 0.25)^2 dx + \int_{0.5}^1 (x - 0.75)^2 dx$$
$$D = 2 \cdot \int_0^{0.5} (x - 0.25)^2 dx = \frac{4}{3} \cdot (0.25)^3 \approx 0.0208$$

Quiz Q1

Q1.3

Now we will use a 2D vector quantizer under MSE distortion which still uses one-bit per symbol. This quantizer takes 2 symbols at a time, represents them as a 2D vector and then uses the optimal 2-bit quantizer.

Which of the following vector quantizer is best in terms of distortion under MSE? Choose all options which have the same lowest expected MSE distortion to get credit.

[ ] {(0.125,0.5), (0.375,0.5), (0.625,0.5), (0.875,0.5)}
[ ] {(0.5,0.125), (0.5,0.375), (0.5,0.625), (0.5,0.875)}
[ ] {(0.25, 0.25), (0.25,0.75), (0.75,0.25), (0.75,0.75)}

Quiz Q1

Q1.4

Which of the two -- scalar or the best 2D quantizer above, is better for lossy compression of uniformly distributed data?
Hint: Remember to compare with respect to bits per symbol.

( ) Scalar
( ) 2D-Vector
( ) Both are equivalent

Performance